

Quantum-Newtonian formulation or of entangled matter

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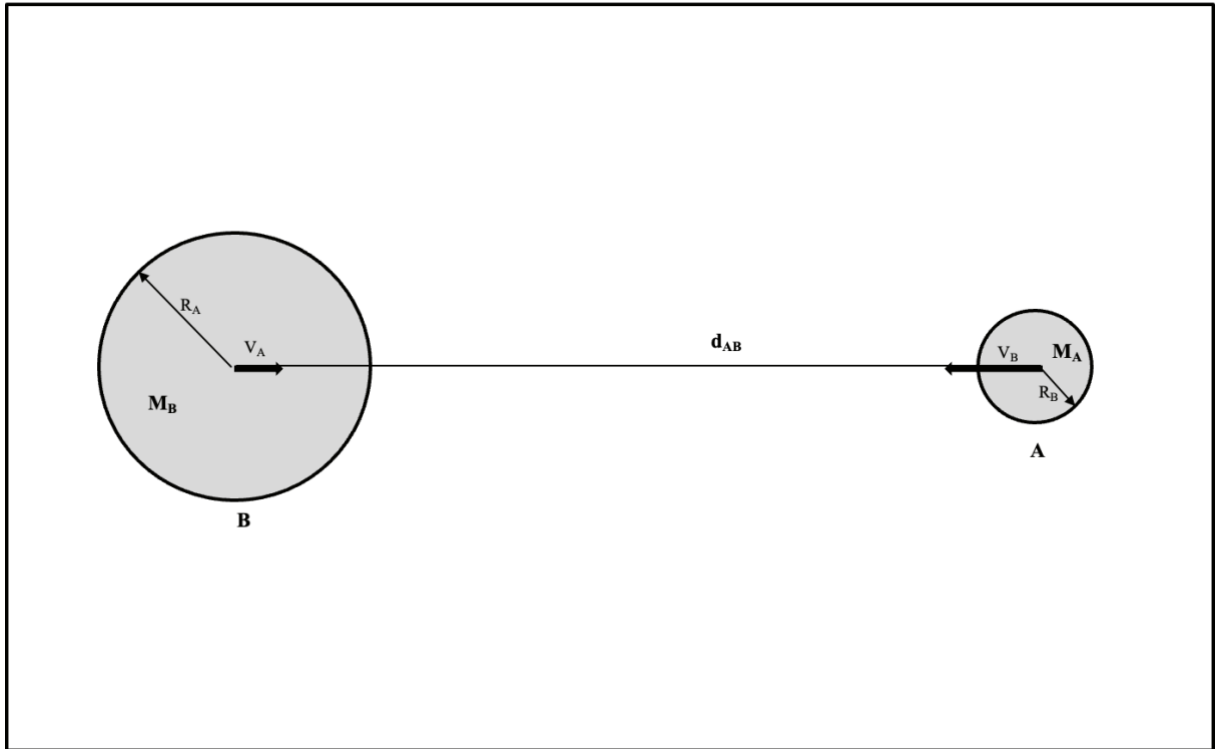
If in the *Quantum-mechanical formulation* We described the quantum origin of the gravitational force and the manifestation as baryonic matter to space in the form of the *Gravitational quantum (from isolated matter)* $\mathfrak{E}$. In this article we describe and detail the gravitational relationship between macroscopic bodies that, for the sake of expositional simplicity, will be spherical, and their evolution over time. With this we address the *Quantum-Newtonian formulation* which will be divided into several headings:

- The gravitational force: Matter finds matter.
- Entangled matter: Vector and quantum-mechanical analysis.
- The gravitational self-vector.

In this way, we introduce a new vision of the Newtonian conception of gravity, without questioning its fundamental concepts, but rather seeking to delve into the nature of the dominant force in our universe.

I. The Gravitational Force: Matter Finds Matter

Figure 5: Gravitory system between two bodys



Let the static spherical bodies A and B in Figure 5 be the masses M_A and M_B , respectively, and d_{AB} the distance separating their centers of mass. The conditions of the environment are the absence of any other baryonic matter and lack of history.

Newton taught us that the gravitational force experienced by both bodies is described by:

$$F_{GAB} = G \frac{M_A \cdot M_B}{d_{AB}^2} \quad \text{Kg} \cdot \text{m} \cdot \text{s}^{-2} \quad (20)$$

Their respective *gravitational vectors* will be:

$$\vec{V}_A = N_A \vec{\mathfrak{E}} \quad \text{m} \cdot \text{s}^{-2} \quad \vec{V}_B = N_B \vec{\mathfrak{E}} \quad \text{m} \cdot \text{s}^{-2} \quad (21)$$

And the gravitational force expressed in terms of *quantum-gravitational theory* is:

$$\vec{F}_{GAB} = \frac{3}{2} \frac{N_A \cdot N_B}{d_{AB}^2} \vec{\mathfrak{E}} \quad \text{eV} \cdot \text{m} \cdot \text{s}^{-2} \quad (22)$$

for $N_B > N_A$.

Expressing (20) and (22) in units of eV and equalizing, we get:

$$G \cdot M_A \cdot M_B = \frac{3}{2} N_A \cdot N_B \cdot \vec{\mathfrak{E}} \quad (23)$$

$$\text{Being } \vec{\mathfrak{E}} = \hat{G} \cdot (E_{PGM} \cdot E_{PGM}) = \hat{G} \hbar^2 \omega^2 \quad (24)$$

There is actually a transformation at the level of their respective quantum:

$$\vec{\mathfrak{E}} \cdot \vec{\mathfrak{E}} = \hat{G} E_{PGM} \cdot \hat{G} E_{PGM} \lesssim \hat{G} (E_{PGM} \cdot E_{PGM}) = \vec{\mathfrak{E}} \quad (25)$$

The relations of (24) and (25) are the manifestation of the *gravitational entanglement* of matter, the expression of which is the *universal gravitational quantum (of entangled matter)*. $\vec{\mathfrak{E}}$ From here on out we will only use this notation. Baryonic matter recognizes each other and becomes gravitationally entangled. Now the new *universal gravitational quantum* takes on the units of energy and takes on a unique character, as it manifests itself in nature.

The concept of *gravitational entanglement* had already been anticipated by Vijay Balasubramanian et al. [17] within the QFT, and later by Sougato Bose et al. [18],[19]. And very recently, entanglement effects have been verified by the ATLAS detector of the LHC [20] and in digital simulations inside an IBM computer by Carlos Sabín [21]. Other contributions come from Marios Christodoulou et al [22]. The application of the quantum concept of entanglement to gravitational attraction is not new, and follows a current trend of trying to give a physical explanation to the origin and nature of the gravitational force. It is enough to remember the contributions that consider it as a force of entropic origin.

This *gravitational entanglement*, whose important evolution over time will be addressed later, the measure of its intensity is the gravitational force due to the mutual attraction between the nucleons of two bodies. In such a way, body A perceives a

gravitational charge N_B from body B and vice versa. And in view of (25), we can already say that it is a quantum coupling constant that plays a key role in entanglement. $\hat{G}$

Explaining the force of gravity using quantum-mechanical ideas such as quantum entanglement with scientific arguments may seem very forced and unscientific, given the evidence that all baryonic matter is attracted. But from the *Quantum-Gravitational Theory* we precisely intend to give a coherent answer to this evidence, why is matter attracted? If we look at similar phenomena in nature, such as the forces of attraction generated between bodies with opposite electric charges or magnetic fields created by materials with these properties, how can we reject a priori that baryonic matter, its nucleons, have a *gravitational charge*? Obviously this argument is more heuristic than scientific, but the relevant relationship of (25) does have a deep scientific significance.

Figure 6				
Gravitatory Study Silver-Gold				
Porton mass(Kg)	$G(m^3/Kg.s^2)$	$\hat{G} (m^3/eV.s^2)$	$E_{PGM}(Ev)$	$\mathcal{E}$
1,672621925950E-27	6,67430000000000E-11	1,78470306952957E-46	6,2551472628667E+08	6,982984013595780E-29
Ag mass (Kg)		Au mass (Kg)		Distance (m)
3		4		1
Nucleons Ag (N_{ag})		Nucleons Au (N_{au})		$Kg \rightarrow eV$
1,793591E+27		2,391455E+27		5,60958860300000E+35
Clasic-Newtonian Formulation (eV)		Quantum-Gravitary Formulation (eV)		
4,492809265560350E+26		4,492809266224940E+26		

Figure 7				
Gravitatory Study Sun-Jupiter				
Proton mass(Kg)	$G(m^3/Kg.s^2)$	$\hat{G} (m^3/eV.s^2)$	$E_{PGM}(Ev)$	$\mathcal{E}$
1,672621925950E-27	6,67430000000000E-11	1,78470306952957E-46	6,2551472628667E+08	6,982984013595780E-29
Sun mass (Kg)		Jupiter mass Kg)		Distance Sun-Jupiter(m)
1,989100E+30		1,898700E+27		7,78412020000000E+11
Sun nucleons (N_s)		Jupiter nucleons (N_j)		$Kg \rightarrow eV$
1,189211E+57		1,135164E+54		5,60958860300000E+35
Clasic-Newtonian Formulation (eV)		Quantum-Gravitary Formulation (eV)		
2,333622676989100E+59		2,333622677334300E+59		

Figures 6 and 7 show two studies considering only the mass of the proton, rather than the average mass of the nucleon. The comparison of the results obtained between classical theory and *quantum-gravitational theory* could not be more conclusive.

With both studies we have demonstrated the validity of the quantum-gravitational formulation for the gravitational force between two bodies. And it is reasonable to think that, if two apparently different formulations obtain the same results, there must be some relationship between the two. And indeed there is as shown below:

$$\begin{aligned}
F_{GAB} &= \frac{3 N_A \cdot N_B}{2 d_{AB}^2} \hat{\mathfrak{E}} = \frac{3 N_A \cdot N_B}{2 d_{AB}^2} \hat{G} \cdot (E_{PGM} \cdot E_{PGM}) \\
&= \frac{3 N_A \cdot N_B}{2 d_{AB}^2} \hat{G} \left(\frac{2}{3} E_{max} \right) \left(\frac{2}{3} E_{max} \right) = \frac{N_A \cdot N_B}{d_{AB}^2} \frac{2}{3} \hat{G} (M_{exp}) (M_{exp}) \\
&= G \cdot \frac{1}{d_{AB}^2} N_A M_{exp} \cdot N_B M_{exp} = G \frac{M_A \cdot M_B}{d_{AB}^2} eV \cdot m \cdot s^{-2} \quad (26)
\end{aligned}$$

As shown, both formulations are equivalent, but their big difference is that in the *quantum-gravitational formulation* the quantum origin of gravity is evident due to the *universal gravitational quantum* $\hat{\mathfrak{E}}$. Let us not forget that And although apparently the two are the same equation expressed in different terms, from the Newtonian equation it is not possible to obtain the quantum-gravitational equation, because the gravitational quantum. it is not a mere product of the quanta of the two bodies, but the gravitational entanglement of both. $E_{PGM} = \hbar \omega \cdot \hat{\mathfrak{E}}$

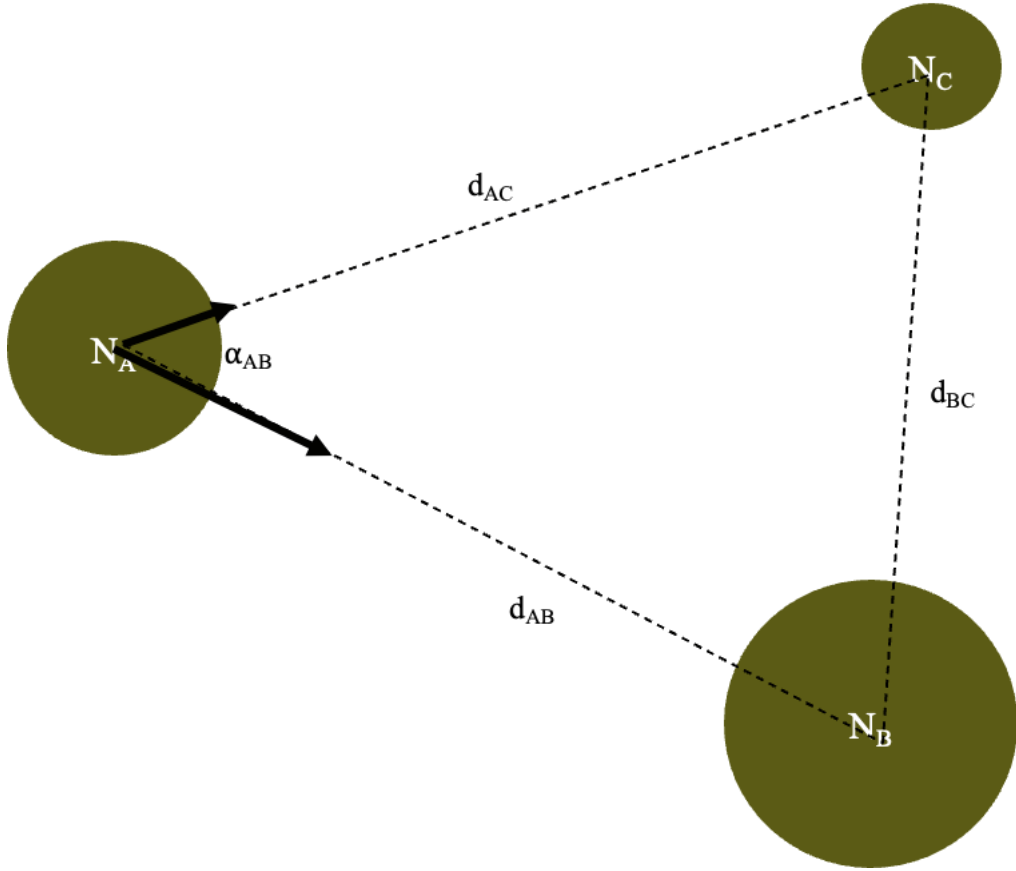
The conditions imposed on the environment are very restrictive but necessary, although they may seem arbitrary. It is about analyzing an ideal situation of two macroscopic bodies that lack history, and in a universe where the only baryonic matter is that provided by them. The necessity is imposed to isolate our laboratory of ideas from the gravitational contamination of our physical life as matter. To do otherwise would force us to go back to the early moments of our universe, just after hadronization, which would divert us from our current goal. However, its discussion and detailed reasoning will be addressed at the end of this chapter.

Finally, the gravitational entanglement shown by the relationship (25) requires a more detailed analysis and in a broader context where matter fills our universe, and considering its evolution over time. First, we will address a primary and instructive vector analysis and then we will perform a quantum-mechanical analysis of its temporal evolution.

II. Entangled Matter: Vector Analysis

Let be an ideal gravitational system consisting of three bodies A, B, and C at any instant t of its dynamical evolution

Figure 8: Quantum-Gravitatory net



The gravitational forces existing in our system will be those generated by the three bodies among themselves:

$$F_{AB} = \frac{3}{2} \cdot \frac{N_A \cdot N_B}{d_{AB}^2} \hat{\mathfrak{E}}; F_{AC} = \frac{3}{2} \cdot \frac{N_A \cdot N_C}{d_{AC}^2} \hat{\mathfrak{E}}; F_{BC} = \frac{3}{2} \cdot \frac{N_B \cdot N_C}{d_{BC}^2} \hat{\mathfrak{E}} \quad (27)$$

The only force exerted on them is the gravitational force, there is no other matter around and we are not interested in their dynamic evolution, or if they are in gravitational equilibrium. We just want to know the resulting gravitational vector on A due to B and C.

If we place our coordinate system on the body A, as shown in Figure 8, the gravitational vectors of B and C will exert their gravitational effects on A dampened by the squares of the distances that separate them.

The resulting gravitational vector will be the vector sum of V_B and V_C :

$$\overrightarrow{V_{BC}} = \sqrt{|V_B|^2 + |V_C|^2 \pm 2 \cdot V_B \cdot V_C \cdot \cos \alpha_{AB}} \quad (28)$$

Or in its developed form

$$\overrightarrow{V_{BC}} = \widehat{\mathfrak{E}} \sqrt{\left| \frac{N_B}{d_{AB}^2} \right|^2 + \left| \frac{N_C}{d_{AC}^2} \right|^2 \pm 2 \cdot \frac{N_B \cdot N_C}{d_{AB}^2 d_{AC}^2} \cdot \cos \alpha_{AB}} \quad (29)$$

It shows that the resulting gravitational vector to which body A is subjected is proportional to the sum of the squares of the gravitational vectors of B and C, applied to A, plus/minus the ratio between the product of the gravitational vectors of B and C, and the product of the distances separating it from A.

Following the same reasoning for a system of n-bodies, we can arrive at a general expression:

$$\overrightarrow{V_R} = \widehat{\mathfrak{E}} \sqrt{\sum_{i=1}^n \left| \frac{N_i}{d_i^2} \right|^2 \pm \sum_{i=1}^n \sum_{j=2}^n B_{ij}} \quad (30)$$

Obviously, the value of V_R will be impossible to obtain if we want to consider all the matter in the universe, which in turn we do not know. We will denote, for simplicity, the modulus of the universal gravitational resultant over any point in space as $\vec{R}$. And now it will be: $\vec{R}$

$$\overrightarrow{V_R} = \widehat{\mathfrak{E}} \vec{R} \quad (31)$$

But we are not interested in the quantization of the resulting V_R vector on a body, but in showing the relevant presence of the *universal gravitational quantum* $\widehat{\mathfrak{E}}$ as a common link to all baryonic matter.

And although the cosmological gravitational result cannot be obtained empirically, we can limit its first terms, which will be the ones that will locally define its value. $\vec{R}$

Vector analysis is exhausted at this point. Regardless of the complexity of the formulation (30), it has achieved its objective by showing that baryonic matter recognizes each other and remains gravitationally entangled by means of the *gravitational quantum* that it itself generates.

III. Entangled Matter: Quantum-Mechanical Analysis

From the combination of formulations (24) and (31) we obtain:

$$\overrightarrow{V_R} = \widehat{\mathcal{E}}\overrightarrow{R} = \hat{G} \hbar^2 \omega^2 \overrightarrow{R} \quad (32)$$

Where it represents the frequency of harmonic-quantum oscillation of the quarks in the nucleons and the gravitational effect of all the matter surrounding the observer. In this more detailed form we can subdivide it for analysis into two parts: the first three terms common to all baryonic matter represent their $\omega\overrightarrow{R}$ *gravitational entanglement*, while the last term represents the *gravitational charge* responsible for the observed effects.

If, as we are proposing, all matter is *gravitationally entangled* in the present moment, it also had to be so since its creation. It was in the process of *hadronization* that the gravitational entanglement and attraction of matter began. Then we have to place our analysis in those first instants of our universe, where the physics of quantum-mechanics was the prevailing one within it.

The history of the universe before the first second was quite chaotic, with an inflationary process and the continuous annihilation between matter and antimatter in the phenomenon known as *baryogenesis*. Therefore, we will be in the lepton era when the universe was only approximately 1 s old and both processes had ended, and the universe was in thermal equilibrium.

Its initial conditions could be summarized as follows:

1. Approximate temperature of 10¹⁰ °K and in thermal equilibrium.
2. A small universe with an expansion factor $a \gg ct$
3. Particle plasma: photons, protons, neutrons, electrons, and neutrinos.
4. Supposed low entropy.
5. Flat space-time.

These conditions define a small, hot, and dense universe. It should be noted that its real size is irrelevant to the study, although it is important to highlight its small volume compared to the current moment. Of all the particles in the primordial plasma, it is the protons and neutrons that actually contribute more than 95% to the density. Photons and neutrinos contribute practically nothing, and electrons account for a tiny part. The relevant fact is that all protons and neutrons were created during the process of hadronization, and from then until today it has remained practically constant. There is no known natural process that generates protons from quarks not contained in previous matter, a necessary condition for the conservation of baryonic number.

The condition of low entropy is imposed by the second law of thermodynamics. The logic of the law tells us that, if entropy always increases in the direction of time, the universe must have less entropy at its beginning than at the present time. Quantifying it in both situations is not an easy or necessary task at the moment. We must be satisfied with logical reasoning, but it will require strong nuances later.

The current estimated density of baryonic matter with respect to the critical density is 0.044. As the estimated critical density is $9.47 \times 10^{-27} \Omega_B \Omega_C \text{ Kg/m}^3$, the density of baryonic matter is $4.1668 \times 10^{-28} \text{ Kg/m}^3$ for an estimated volume of observable universe of about 43,500 106 light-years. The baryonic density corresponding to a universe of 4 light-years is $4.5314 \times 10^{-18} \text{ Kg/m}^3$, which is much higher than that estimated for that era of the universe. This density corresponds to approximately $2.7 \times 10^{18} \text{ protons/cm}^3$. The accelerated expansion may have contributed to a sharp decrease in the density of matter, but even so in that first second we can idealize our universe as a ball of radiation and matter in the form of protons and neutrons.

And in this idealized context of an expanding volume populated by protons and neutrons (the rest of the particles do not significantly influence) with an isotropic and uniform distribution, we can disregard the term B_{ij} from equation (30), and since $N_i = 1$, we obtain that for a nucleon in located at its center the baryonic attraction exerted by the rest is:

$$\vec{V}_R = \hat{\mathbb{G}} \sqrt{\sum_{i=1}^N \left| \frac{1}{d_i^2} \right|^2} \quad (33)$$

This extreme idealization is another temporal perspective of (32), where now $\mathbf{R}$ does not represent the grouped matter, but each nucleon individually, a step prior to the entire subsequent gravitational process. Baryonic matter performs an exchange of quantum information simultaneous to its own creation, sharing its *gravitational charge* indefinitely. This exchange of quantum information can be assimilated to the *exchange interaction* described for magnetism by the Heitler-London theory.

Sougato Bose et al [18] describe the lowest level of *gravitational entanglement* between two quantum-harmonic oscillators as:

$$\Delta\phi = \frac{2Gm^2}{d^3} x_A x_B \quad (34)$$

Its equivalent in the GCT would be:

$$\Delta\phi = 3\hat{\mathbb{G}} \frac{N_A N_B}{d^3} x_A x_B = 3\hat{G} \hbar^2 \omega^2 \frac{N_A N_B}{d^3} x_A x_B \quad (35)$$

Equation (15) is the third term in a series obtained from two quantum-harmonic oscillators of equal masses. Our reasoning is that this term is equally valid for unequal masses. In (35) the constant plays the role of $(N_A \neq N_B) \hat{G} a$ *gravitational entanglement constant* that we already intuited in (24), thus acquiring a profound physical meaning.

The physical picture we are trying to describe is that of a small but expanding universe, with a very high density of protons and neutrons endowed with great kinetic energy, carriers of *gravitational charges* that produce their global entanglement. It is the description of a quantum universe that gravity later also made classical, although it has not ceased to be quantum. The gravitational force is quantum, as shown by *gravitational entanglement*, but its effects observed by classical entities are obviously classical, and therefore are perceived by our senses.

The physical description of the temporal evolution from that moment 0 to the present is a scientific challenge that has not yet been resolved. Evolving from a boiling

plasma of protons and neutrons to mass structures such as stars and black holes is impossible without a primary attraction of the fundamental constituents of matter, regardless of whether other physical phenomena contributed. Let's not forget the electrical repulsion between protons itself, which is on the order of 36 times greater than their gravitational attraction.

The process of atomic formation brought about electrical neutrality, eliminated the consequent repulsion, and contributed considerably to gravitational evolution, but it could not have been enough to create such enormous masses in such a short space of time, as the Webb telescope is showing us [23][24]. If we add to this the fifth initial condition of a flat universe, since the homogeneous and isotropic plasma of protons and neutrons could not distort the geometry of space-time, we find ourselves in a utopian situation of immense and diluted HII clouds, which on their own would never have collapsed in a continuously expanding universe. Despite the rapid cooling of the universe, the weak *gravitational entanglement* was not enough to cause the collapse of matter as we know it today. There had to be some kind of space-time instabilities for that universe to stop being so homogeneous and uniform, and to trigger all the great gravitational evolution of matter.

There are currently three main hypotheses in this line of argument: *Dark energy*; *Dark matter* and *Primordial Black Holes* (NPOs). Our ideas are in support of the PNAs, as we will argue and develop in depth in the chapter dedicated to the *quantum-relativistic formulation*.

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